EXERCISE BAT2: COMPOSITE BEAM - SOLUTIONS

Problem 1: Question 1.1

Lateral torsional buckling (LTB) prevented by the steel sheeting.
Section classification, SZS C5, class 2 for n = 1.0, so no limit to this class even under M+N → EP calculation. Note : satisfies even class 1 requ.
Construction stage, structural safety (EE):
For this stage, limitation of the elastic resistance of the steel beam.
$M_{el,\,Rd} = 235 \cdot 713000/1.05 = 159.6$ kNm
$V_{el,\,Rd} = 135 \cdot 2390/1.05 = 308.8$ kN
Note: we can also, conservatively, use the strength of materials formula: $V_{el,Rd} = \frac{\tau_{yd} \cdot I_y \cdot t_w}{S_y} = 282\ kN$

Checks:
$M_{Ed} = 131.4$ kNm < $M_{el,\,Rd} = 159.6$ kNm OK
$V_{Ed} = 58.8$ kN < $V_{el,\,Rd} = 136 * 2390/1.05 = 308.8$ kN OK.

No M-V interaction because it is a single span beam under uniform loads.

Problem 1: Question 1.2

Final stage, structural safety:
Strength of the mixed section of the 2nd beam:
Effective width (*SIA 262 4.1.3.3 and TGC 10 p.209)*
$b_{eff,i} = 0.2\ (2000\text{-}160)/2 + 0.1 * 10000 = 1184$ mm < 0.2 l0 = 2000 mm
$b_{eff} = 2\ b_{eff,i} + b_w = 2 * 1184 + 160 = 2528$ mm > 2000 mm = a → $b_{eff} = 2000$ mm

$$b_{eff,1} = 0.2 \cdot b_1 + 0.1 \cdot l_0 \leq 0.2 \cdot l_0$$

Plastic equivalence coefficient (*TGC 10 p.148)*

$$n_{pl} = \frac{f_y \cdot \gamma_c}{0.85 \cdot f_{ck} \cdot \gamma_a} = \frac{235 N/mm^2 \cdot 1.5}{0.85 \cdot 25 N/mm^2 \cdot 1.05} = 15.8$$

Neutral axis position (TGC 10, p.163, table 4.60, or TGC 11, table 10.23) :
Hypothesis, the reinforcing bars are neglected.

$A_a = 6260\ mm^2 < A_c/n_{pl} = 2000 \cdot (100\text{-}40)/15.8 = 7595\ mm^2$ → plastic neutral axis in the slab
Additional calculation: $z_{pl,b} = h - n_{pl} \cdot A_a/b_{eff} = 430 - 15.8 \cdot 6260/2000 = 380.5$ mm (> 330 + 40, indeed in the slab)

Plastic bending resistance

$$W^+_{pl,b} = A_a \cdot (h - z_a - 0.5 \cdot n_{pl} \frac{A_a}{b_{eff}})$$

$$W^+_{pl,b} = 6260 \left(430 - 330/2 - 0.5 \cdot 15.8 \frac{6260}{2000}\right) = 1504 \cdot 10^3\ mm^3$$

$$M_{pl,b,Rd} = \frac{235 \cdot 1504 \cdot 10^3}{1.05} = 336.6\ \text{kNm}$$

Checks:

$M_{Ed} = 269.2$ kNm < $M_{pl,\,b,Rd} = 336.6$ kNm OK
$V_{Ed} = 104.2$ kN < $V_{pl,\,Rd} = 398$ kN OK. No M-V interaction, single span beam

Remarks:

- If the verification was not satisfied, consideration could be given to the installation of a temporary prop at mid-span.
- The steel sheeting is considered to prevent the beams LTB.
- In reality, according to SIA code, the erection load applies to a maximum area of 3m x 3m. Here, for simplicity of calculation, a more unfavourable load case is considered.

Problème 2 : Question 1.1

Section classification and LTB :

- Slenderness criteria at construction stage

Aile (compression) : $\frac{c}{t_f} = \frac{b - t_{w-2r}}{2t_f} = \frac{280 \text{ mm} - 8 \text{ mm} - 2 \cdot 24 \text{ mm}}{2 \cdot 13 \text{ mm}} = 8.6 < 9\varepsilon = 9 \Rightarrow$ classe de section 1

Ame (flexion) : $\frac{b_w}{t_w} = \frac{h_1}{t_w} = \frac{196 \text{ mm}}{8 \text{ mm}} = 24.5 < 72\varepsilon = 72 \Rightarrow$ classe de section 1

- Critères d'élancement au stade définitif

Aile (compression) : $\frac{c}{t_f} = 8.6 < 9\varepsilon = 9 \Rightarrow$ classe de section 1

Ame (compression) : Cas défavorable, considérée comme entièrement comprimée

$\frac{b_w}{t_w} = 24.5 < 33\varepsilon = 33 \Rightarrow$ classe de section 1

Note : Selon la SZS C5/05, la section est de classe 1 pour $\frac{N_{Ed}}{N_{Rd}} = 1.0$.

- Déversement au stade de construction

Le sommier est tenu latéralement tous les 2 m par les solives : $l_D = 2000$ mm.
Cas défavorable : $\psi = 1.0$

$$l_{cr} = 2.7 i_z (1 - 0.5\psi)\sqrt{\frac{E}{f_{ya}}} = 2.7 \cdot 70 \text{ mm}(1 - 0.5 \cdot 1.0)\sqrt{\frac{210\,000 \text{ N/mm}^2}{235 \text{ N/mm}^2}} = 2825 \text{ mm}$$

Comme $l_D = 2000$ mm $< 1.1 \cdot l_{cr} = 1.1 \cdot 2825$ mm $= 3107$ mm, le déversement n'est pas déterminant.

- Déversement au stade définitif

Vérification pas nécessaire car : $h_{HEA280} = 270$ mm < 800 mm
$h_{dalle} = 100$ mm
connexion par goujons à tête
raidisseur sur appui

Problem 2: Question 1.2

Construction stage, determination of internal forces for the load cases defined in the data:

$$\begin{aligned} M_{Ed}^{+} &= 1.35\,[0.08\,(g_a + g_p) + 0.0939\,g_c]l^2 + 1.50 \cdot 0.0939\,q_m l^2 \\ &= 1.35\,[0.08\,(2.2 \text{ kN/m} + 0.8 \text{ kN/m}) + 0.0939 \cdot 18 \text{ kN/m}](6 \text{ m})^2 \\ &+ 1.50 \cdot 0.0939 \cdot 2 \text{ kN/m} \cdot (6 \text{ m})^2 = 104 \text{ kNm} \end{aligned}$$

$$\begin{aligned} M_{Ed}^{-} &= 1.35\,[-0.1\,(g_a + g_p) - 0.1167\,g_c]l^2 + 1.50(-0.1167)\,q_m l^2 \\ &= 1.35\,[-0.1\,(2.2 \text{ kN/m} + 0.8 \text{ kN/m}) - 0.1167 \cdot 18 \text{ kN/m}](6 \text{ m})^2 \\ &+ 1.50 \cdot (-0.1167) \cdot 2 \text{ kN/m} \cdot (6 \text{ m})^2 = -129 \text{ kNm} \end{aligned}$$

$$\begin{aligned} V_{Ed} &= 1.35\,[0.6\,(g_a + g_p) + 0.617\,g_c]l + 1.50 \cdot 0.617\,q_m l \\ &= 1.35\,[0.6\,(2.2 \text{ kN/m} + 0.8 \text{ kN/m}) + 0.617 \cdot 18 \text{ kN/m}]6 \text{ m} \\ &+ 1.50 \cdot 0.617 \cdot 2 \text{ kN/m} \cdot 6 \text{ m} = 116 \text{ kN} \end{aligned}$$

Structural safety, construction stage (EE method)

The HEA 280 (S 235) profile meets the requirements of section class 3 and the steel sheeting prevent its LTB.

– Cross-sectional strength (cross-section characteristics from SZS C5):

$M_{el,Rd} = 1010 \cdot 10^3 \cdot 235/10^6 = 237.3$ kNm

$V_{el,Rd} = 2060 \cdot 135/10^3 = 278.1$ kN

– Checks:

$M_{Ed}^{-} = 129$ kNm < 226 kNm= $M_{el}/\gamma_a = 237.3/1.05$ OK

$V_{Ed} = 116$ kN < 264.8 kN = $V_{el}/\gamma_a = 278.1/1.05$ OK

No interaction to check as $V_{Ed} < 50\%\ V_{Rd}$

<u>Problem 2</u>: Question 1.3

Definitive stage (EP), determination of the loads positions, then of the internal forces (we use the SZS C4)

We could redistribute 40% because Class 1 (do not confuse class and calculation method, here EP):

Le calcul des efforts intérieurs est effectué selon la méthode élastique-plastique d'une poutre à inertie constante (section non fissurée). On peut donc admettre une redistribution des moments sur appui de 30%

– Charge permanente :

$$g_{Ed} = \gamma_G (g_a + g_p + g_c + g_{fin}) = 1.35\,(2.2\text{ kN/m} + 0.8\text{ kN/m} + 18\text{ kN/m} + 12.8\text{ kN/m}) = 45.6\text{ kN/m}$$

Moments élastiques

$$M_{g1,Ed} = 0.08\, g_{Ed} l^2 = 0.08 \cdot 45.6\text{ kN/m} \cdot (6\text{ m})^2 = 131\text{ kNm}$$
$$M_{g2,Ed} = 0.025\, g_{Ed} l^2 = 0.025 \cdot 45.6\text{ kN/m} \cdot (6\text{ m})^2 = 41\text{ kNm}$$
$$M_{gB,Ed} = -0.1\, g_{Ed} l^2 = -0.1 \cdot 45.6\text{ kN/m} \cdot (6\text{ m})^2 = -164\text{ kNm}$$

Moments redistribués (fig. 10.60)

$$M_{gB,Ed} = (1-0.3)\cdot(\cdot \qquad , \qquad 115\text{ kNm}$$
$$M_{g2,Ed} = 41\text{ kNm} + 0.3 \cdot 164\text{ kNm} = 90.2\text{ kNm}$$
$$M_{g1,Ed} = -\frac{g_{Ed} x^2}{2} + R_{A,Ed}\, x = -\frac{45.6\text{ kN/m} \cdot (2.55\text{ m})^2}{2} + 118\text{ kN} \cdot 2.55\text{ m} = 153\text{ kNm}$$

avec

$$V_{A,d} = R_{A,Ed} = \frac{-M_{B,Ed}}{l} + \frac{g_{Ed}\, l}{2} = \frac{-115\text{ kNm}}{6\text{ m}} + \frac{45.6\text{ kN/m} \cdot 6\text{ m}}{2} = 118\text{ kN}$$
$$V_{B,g} = R_{B,Ed} = \frac{M_{B,Ed}}{l} + \frac{g_{Ed}\, l}{2} = \frac{115\text{ kNm}}{6\text{ m}} + \frac{45.6\text{ kN/m} \cdot 6\text{ m}}{2} = 160\text{ kN}$$

Position du point de moment maximal (effort tranchant nul)

$$x = \frac{R_{A,Ed}}{R_{A,Ed} + R_{B,Ed}} \cdot l = \frac{118\text{ kNm}}{118\text{ kNm} + 160\text{ kNm}} \cdot 6\text{ m} = 2.55\text{ m}$$

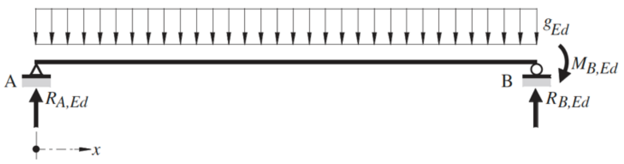


Figure 2.1 : Redistribution of moments on the edge span

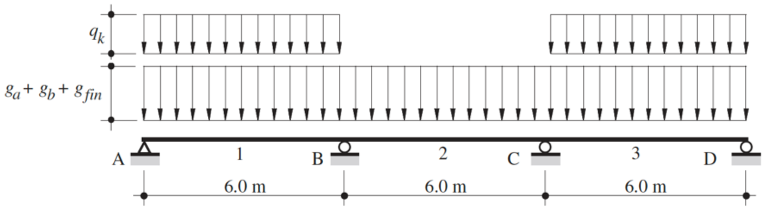
– Charge utile :

$q_{k,Ed} = \gamma_Q q_k = 1.50 \cdot 24\text{ kN/m} = 36\text{ kN/m}$

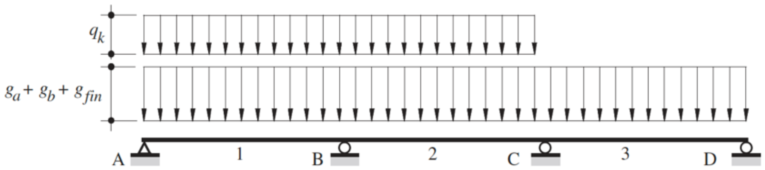
La position des charges provoquant le moment maximal dans la première travée est illustrée à la Figure 2.2a

Moments élastiques

$$M_{qB,Ed} = -0.05 q_{k,Ed} l^2 = -0.05 \cdot 36\text{ kN/m} \cdot (6\text{ m})^2 = -64.8\text{ kNm}$$
$$M_{q1,Ed} = 0.1013 q_{k,Ed} l^2 = 0.1013 \cdot 36\text{ kN/m} \cdot (6\text{ m})^2 = 131\text{ kNm}$$



(a) Moment positif maximal



(b) Moment négatif et effort tranchant maximaux

Figure 2.2 : Positioning of loads to obtain maximum forces at the final stage.

Moments redistribués

$$M_{qB,Ed} = (1 - 0.3) \cdot (-64.8 \text{ kNm}) = -45.4 \text{ kNm}$$

$$M_{q1,Ed} = \frac{-q_{k,Ed}\, x^2}{2} + R_{A,Ed}\, x = \frac{-36 \text{ kN/m} \cdot (2.78 \text{ m})^2}{2} + 100 \text{ kN} \cdot 2.78 \text{ m} = 139 \text{ kN}$$

avec

$$V_{A,d} = R_{A,Ed} = \frac{-M_{qB,Ed}}{l} + \frac{q_{k,Ed}\, l}{2} = \frac{-45.4 \text{ kNm}}{6 \text{ m}} + \frac{36 \text{ kN/m} \cdot 6 \text{ m}}{2} = 100 \text{ kN}$$

$$V_{B,g} = R_{B,Ed} = \frac{M_{qB,Ed}}{l} + \frac{q_{k,Ed}\, l}{2} = \frac{45.4 \text{ kNm}}{6 \text{ m}} + \frac{36 \text{ kN/m} \cdot 6 \text{ m}}{2} = 116 \text{ kN}$$

$$x = \frac{R_{A,Ed}}{R_{A,Ed} + R_{B,Ed}}\, l = \frac{100 \text{ kNm}}{100 \text{ kNm} + 116 \text{ kNm}} \cdot 6 \text{ m} = 2.78 \text{ m}$$

La position des charges provoquant le moment maximal sur appui est illustrée à la Figure 2.2b

Moment élastique

$$M_{qB,Ed} = -0.1167 q_{k,Ed} l^2 = -0.1167 \cdot 36 \text{ kN/m} \cdot (6 \text{ m})^2 = -151 \text{ kNm}$$

Moment redistribué

$$M_{qB,Ed} = (1 - 0.3) \cdot (-151 \text{ kNm}) = -106 \text{ kNm}$$

$$V_{B,g} = R_{B,Ed} = \frac{M_{qB,Ed}}{l} + \frac{q_{k,Ed}\, l}{2} = \frac{106 \text{ kNm}}{6 \text{ m}} + \frac{36 \text{ kN/m} \cdot 6 \text{ m}}{2} = 126 \text{ kN}$$

– Valeurs de calcul des efforts intérieurs (charge utile prépondérante) :
  En travée

$$M^+_{Ed} = M_{g1,Ed} + M_{q1,Ed} = 153 \text{ kNm} + 139 \text{ kNm} = 292 \text{ kNm}$$

Sur appui

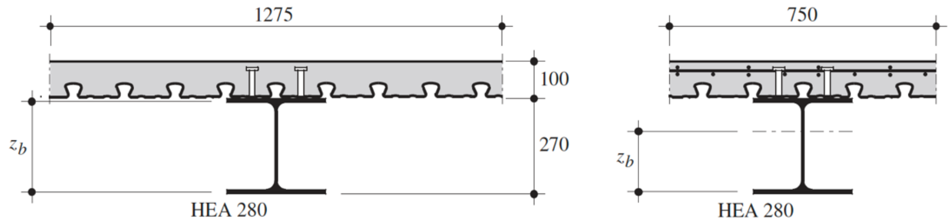
$$M^-_{Ed} = M_{g1,Ed} + M_{q1,Ed} = -115 \text{ kNm} - 106 \text{ kNm} = -221 \text{ kNm}$$

$$V_{Ed} = 160 \text{ kN} + 126 \text{ kN} = 286 \text{ kN}$$

Problem 2: Question 1.4

- Stade définitif (méthode EP)
  - Résistance des sections en travée de rive :
    Largeur participante (Figure 2.3a) :
    Dans cet exemple, nous utilisons la largeur participante $b_{eff,1}$ pour la première travée définie dans la norme EN 1994-1-1 ainsi que ses notations :
    $b_{eff,1} = b_0 + \sum b_{ei} = b_{e1} + b_{e2} = 2b_e$
    $b_{ei} = L_e/8 \leq b_i$
    avec $b_0 = 0$ (structure de bâtiment)
    $L_e = 0.85\, L_1 = 0.85 \cdot 6000 \text{ mm} = 5100 \text{ mm}$
    $b_i = 8000 \text{ mm}/2 = 4000 \text{ mm}$

    $b_{eff,1} = 2b_e = 2 \cdot L_e/8 = 2 \cdot 5100 \text{ mm}/8 = 1275 \text{ mm}$



(a) Section en travée de rive (b) Section sur appui

Figure 2.3 : Cross-sections of the continuous mixed beam in span and on supports.

Note : Le TGC vol. 10 simplifie et ne donne que le cas de la poutre simple, où $b_{eff} = \frac{L_0}{4} < a$. Dans le cas de cet exemple, on a $b_{eff} = 0.85 \cdot \frac{L_0}{4} < a$.

$A_c = h_{eq} b_{eff,1} = 86.4 \text{ mm} \cdot 1275 \text{ mm} = 110 \cdot 10^3 \text{ mm}^2$

Position de l'axe neutre

$$A_a = 9730 \text{ mm}^2 > \frac{A_c}{n_{pl}} = \frac{110 \cdot 10^3 \text{ mm}^2}{15.8} = 6962 \text{ mm}^2$$

$$\text{avec } n_{pl} = \frac{f_{ya}}{0.85\, f_{ck}} \cdot \frac{\gamma_c}{\gamma_a} = \frac{235 \text{ N/mm}^2}{0.85 \cdot 25 \text{ N/mm}^2} \cdot \frac{1.5}{1.05} = 15.8$$

⇒ L'axe neutre se situe dans le profilé. La tôle, comprimée, est négligée.

$$A_a - 2\, b\, t_f = 9730 \text{ mm}^2 - 2 \cdot 280 \text{ mm} \cdot 13 \text{ mm} = 2450 \text{ mm}^2 < \frac{A_c}{n_{pl}} = 6962 \text{ mm}^2$$

⇒ L'axe neutre se situe dans l'aile supérieure du profilé.

Position de l'axe neutre

$$z_b = h_a + \frac{1}{2b}\left(\frac{A_c}{n_{pl}} - A_a\right) = 270 \text{ mm} + \frac{1}{2 \cdot 280 \text{ mm}} \cdot \left(\frac{110 \cdot 10^3 \text{ mm}^2}{15.8} - 9730 \text{ mm}^2\right) = 265 \text{ mm}$$

Module de section plastique

$$\begin{aligned} W^+_{pl,b} &= \frac{A_c}{n_{pl}} z_c - A_a\, z_a + b(h_a^2 - z_b^2) \\ &= \frac{110 \cdot 10^3 \text{ mm}^2}{15.8} \cdot \left(270 \text{ mm} + 100 \text{ mm} - \frac{86.4 \text{ mm}}{2}\right) - 9730 \text{ mm}^2 \cdot \frac{270 \text{ mm}}{2} \\ &+ 280 \text{ mm} \cdot \left[(270 \text{ mm})^2 - (265 \text{ mm})^2\right] = 1714 \cdot 10^3 \text{ mm}^3 \end{aligned}$$

Valeur de calcul du moment résistant plastique

$$M^+_{pl,Rd} = \frac{f_{ya} W^+_{pl,b}}{\gamma_a} = \frac{235 \text{ N/mm}^2 \cdot 1714 \cdot 10^3 \text{ mm}^3}{1.05} = 384 \cdot 10^6 \text{ Nmm} = 384 \text{ kNm}$$

- Résistance des sections sur appui :
  Largeur participante $b_{eff,2}$ pour la travée centrale selon la norme EN 1994-1-1 (Figure 2.3b) :
  $L_e = 0.25(L_1 + L_2) = 0.25 \cdot 2 \cdot 6000 \text{ m} = 3000 \text{ mm}$
  $b_{eff,2} = 2 \cdot b_e = 2 \cdot \dfrac{L_e}{8} = 2 \cdot \dfrac{3000 \text{ mm}}{8} = 750 \text{ mm}$
  $A_{ct} = h_{eq}\, b_{eff,2} = 86.4 \text{ mm} \cdot 750 \text{ mm} = 64.8 \cdot 10^3 \text{ mm}^2$

  Choix de l'armature Ø 10 mm, $s_f = 200$ mm, $A_s = 393$ mm²/m :

  Position de l'axe neutre (tab. 10.23)

  $$n_{pls} = \frac{f_{yd}}{f_{sd}} = \frac{235 \text{ N/mm}^2}{500 \text{ N/mm}^2} \cdot \frac{1.15}{1.05} = 0.51$$

  $A_s = 393 \text{ mm}^2/\text{m} \cdot 0.75 \text{ m} = 295 \text{ mm}^2/\text{m}$
  $A_a = 2b\, t_f + (h_a - 2t_f)\, t_w = 2 \cdot 280 \text{ mm} \cdot 13 \text{ mm} + (270 \text{ mm} - 2 \cdot 13 \text{ mm}) \cdot 8 \text{ mm} = 9232 \text{ mm}^2$

  $A_a - 2b\, t_f = 9232 \text{ mm}^2 - 2 \cdot 280 \text{ mm} \cdot 13 \text{ mm} = 1952 \text{ mm}^2$

  $$\frac{A_s}{n_{pls}} = \frac{295 \text{ mm}^2}{0.51} = 578 \text{ mm}^2$$

  $$A_a - 2b\, t_f > \frac{A_s}{n_{pls}} \Rightarrow 1952 \text{ mm}^2 > 578 \text{ mm}^2 \Rightarrow \text{axe neutre dans l'âme du profilé}$$

Position de l'axe neutre

$$\begin{aligned} z_b &= \frac{1}{2\, t_w}\left(\frac{A_s}{n_{pls}} - A_a\right) + \frac{b t_f}{t_w} + h_a - t_f \\ &= \frac{1}{2 \cdot 8 \text{ mm}} \cdot (578 \text{ mm}^2 - 9232 \text{ mm}^2) + \frac{280 \text{ mm} \cdot 13 \text{ mm}}{8 \text{ mm}} + 270 \text{ mm} - 13 \text{ mm} = 171 \text{ mm} \end{aligned}$$

Module de section plastique

$$\begin{aligned} W^-_{pl,b} &= \frac{A_s}{n_{pls}} z_s - A_a \cdot z_a + 2b\, t_f\left(h_a - \frac{t_f}{2}\right) + t_w\, (h_a - t_f)^2 - t_w\, z_b^2 \\ &= 578 \text{ mm}^2 \cdot \left(370 \text{ mm} - \frac{60 \text{ mm}}{2}\right) - 9232 \text{ mm}^2 \cdot \frac{270 \text{ mm}}{2} + 2 \cdot 280 \text{ mm} \cdot 13 \text{ mm} \\ &\cdot \left(270 \text{ mm} - \frac{13 \text{ mm}}{2}\right) + 8 \text{ mm}\, (270 \text{ mm} - 13 \text{ mm})^2 - 8 \text{ mm} \cdot (171 \text{ mm})^2 \\ &= 1163 \cdot 10^3 \text{ mm}^3 \end{aligned}$$

Valeur de calcul du moment résistant plastique

$$M^-_{pl,Rd} = \frac{f_{yd}\, W^-_{pl,B}}{\gamma_a} = \frac{235 \text{ N/mm}^2 \cdot 1163 \cdot 10^3 \text{ mm}^3}{1.05} = 260 \cdot 10^6 \text{ Nmm} = 260 \text{ kNm}$$

- Vérifications :
  A mi-travée :

  $M_{Ed}^{+} = 292\ \text{kNm} < M_{Rd}^{+} = \ \ 384\ \text{kNm}$ OK

  Sur appui :

  $M_{Ed}^{-} = 221\ \text{kNm} < M_{pl,Rd}^{-} = 260\ \text{kNm}$ OK

  $V_{Ed} = 286\ \text{kN} < V_{pl,Rd} = 410\ \text{kN}$ OK

  Interaction M-V :

  $V_{Ed} = 286\ \text{kN} > 50\%\ V_{Ed} = 205\ \text{kN} \Rightarrow$ l'interaction M-V doit être vérifiée.

$$M_{V,Rd} = \frac{b \cdot t_f \cdot f_y \cdot (h - t_f)}{\gamma_{M1}} + \frac{h_2^2 \cdot t_w \cdot f_y}{4 \cdot \gamma_{M1}} \cdot \left[1 - \left(\frac{V_{Ed}}{V_{Rd}}\right)^2\right]$$
$$= \frac{280\ \text{mm} \cdot 13\ \text{mm} \cdot 235\ \text{N/mm}^2 \cdot (270\ \text{mm} - 13\ \text{mm})}{1.05}$$
$$+ \frac{(244\ \text{mm})^2 \cdot 8\ \text{mm} \cdot 235\ \text{N/mm}^2}{4 \cdot 1.05} \cdot \left[1 - \left(\frac{286\ \text{kN}}{410\ \text{kN}}\right)^2\right]$$
$$= 223 \cdot 10^6\ \text{Nmm} = 223\ \text{kN}$$

  Vérification :

  $M_{Ed}^{-} = 221\ \text{kN} < M_{V,Ed} = 223\ \text{kN}$ OK

<u>Problem 2</u>: Question 1.5, SLS

Assumption: In this exercise, the deflections are determined elastically using a constant inertia along the beam (Inertia of the composite section in span "I+")

- Stade de construction
  Il s'agit de vérifier la flèche de la poutre métallique sous l'effet des charges permanentes (cas de charge quasi permanent).
  Flèche due au poids propre des profilés g_a et de la dalle $(g_p + g_c)$:

$$w(g_a + g_p + g_c) = \frac{32.2 \cdot (g_a + g_p + g_c)\ l^4}{I_{ya}} = \frac{32.2 \cdot (2.2 + 0.8 + 18) \cdot 10^3 \cdot (6\ \text{m})^4}{136.7 \cdot 10^6\ \text{mm}^4} = 6.4\ \text{mm}$$

  Vérification :

  $w(g_a + g_p + g_c) = 6.4\ \text{mm} < \frac{l}{300} = \frac{6000}{300} = 20\ \text{mm}$ OK

- Stade définitif
  Il s'agit de vérifier la flèche de la poutre mixte sous l'effet de cas de charge fréquent et quasi permanent. Les actions à considérer sont les charges permanentes, y compris les finitions, la charge utile ainsi que les effets à long terme dus au retrait et au fluage du béton.

Les inerties sont calculées à l'aide des formules du tableau 10.22. La détermination des flèches au stade définitif est effectuée à l'aide de la SZS C4/06. Les principales grandeurs sont résumées dans le tableau

Tableau 2.1: Résumé des principales grandeurs pour le calcul des flèches.

	Poids des finitions g_{fin}	Charge utile		Retrait
		Courte durée	Longue durée	
g, q [kN/m]	12.8	24	24	
n	18.3	6.1	18.3	12.2
I_b^- $[10^6\ \text{mm}^4]$	149	149	149	149
I_b^+ $[10^6\ \text{mm}^4]$	277	380	277	315
I_b^- / I_b^+	0.55	0.39	0.55	0.48

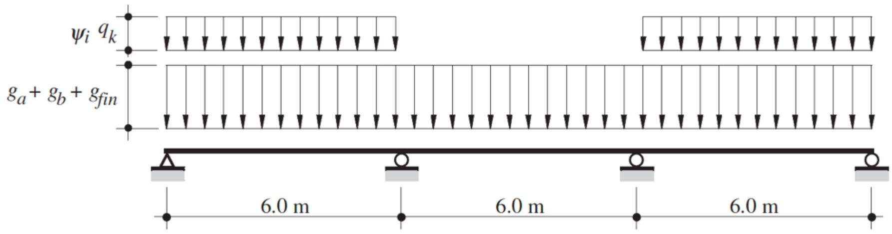


Figure 2.4: Positioning of loads for the calculation of deflections.

Checking deflections:

- Frequent load case, suitability for operation

$$w = w_2 + w_{31} \ \leq \ \frac{l}{350}$$
$$w = w\left(g_{fin}\right) + \ w_s \ + \psi_1 w(q_k)_{court}$$
$$\boldsymbol{w\left(g_{fin}\right)} = \frac{32.2 \cdot \ g_{fin} \cdot l^4}{I^+_{b\varphi}} = \frac{32.2 \cdot 12.8 \ \cdot \ 10^3 \frac{N}{m} \cdot (6m)^4}{269 \ \cdot \ 10^6 \ mm^4} = 2.0 \ mm$$

$$\boldsymbol{w_s} = \frac{M_{cs} l^2}{20 E I^+_{bs}} = \frac{95 \ \cdot \ 10^6 Nmm \ \cdot (6000 \ mm)^2}{20 \cdot 210 \cdot 10^3 N/mm^2 \cdot 308 \cdot 10^6 mm^4} = 2.6 \ mm$$

Avec :
$$M_{cs} = \ N_{cs}(z_c - z_{bs}) = 947 \ kN \ \cdot (0.327 \ m - 0.227 \ m) = 95 \ kNm$$
$$N_{cs} = \varepsilon_{cs,\infty} E_{cs} A_c = 0.50 \cdot 10^{-3} \cdot 17.2 \frac{kN}{mm^2} \cdot 1275 mm \cdot 86.4 mm$$
$$z_c = 370 mm - \frac{86.4 \ mm}{2} = 327 mm$$
$$\boldsymbol{\psi_1 w(q_k)_{court}} = 0.5 \ \cdot \frac{47.1 q_k l^4}{I^+_{b0}} = \frac{47.1 \cdot 24 \cdot \frac{10^3 N}{m} \cdot (6m)^4}{377 \cdot 10^6 mm^4} = 0.5 \cdot 3.9 \ mm = 2.0 \ mm$$

$$\boldsymbol{w = 2.0 \ mm + \ 2.6 \ mm \ + 2.0 \ mm = 6.6 \ mm \ \leq \ \frac{l}{350} = 17.1 \ mm}$$

Cas de charge fréquent, utilisation et exploitation, confort

$$w = \psi_1 w(q_k)_{court} = 0.5 \cdot 3.9 \text{ mm} = 2.0 \text{ mm} < \frac{l}{350} = 17.1 \text{ mm}$$

Cas de charge quasi permanent, aspect

$$w = w(g_a + g_b) + w(g_{fin}) + w_s + \psi_2 w(q_k)_{long} \leq \frac{l}{300}$$
$$w = 6.4 \text{ mm} + 2.0 \text{ mm} + 2.6 \text{ mm} + 0.3 \cdot 5.4 \text{ mm}$$
$$= 12.6 \text{ mm} < \frac{6000 \text{ mm}}{300} = 20 \text{ mm}$$

According to SIA 264, the SLS check must be carried out taking into account the various effects set out in § 5.1.3.1 . However, it becomes difficult to account for all these effects when calculating by hand

5.1.3 **Aptitude au service**

5.1.3.1 Le calcul des contraintes et des déformations à l'état-limite d'aptitude au service doit, le cas échéant, prendre en compte les aspects suivants :
– largeur participante (traînage de cisaillement)
– glissement provenant d'une connexion partielle importante et/ou soulèvement
– fissuration de la dalle en béton dans les zones de moments négatifs
– fluage et retrait du béton
– plastification éventuelle de l'acier de construction ou de l'acier d'armature.

Appendix: Table 2.1 Inertia Calculation

Short load duration: $n_{el,\,short} = 6.1$

Position of the neutral axis:

$$A_a(h - z_a - h_c) \geq \frac{A_c}{n_{el,court}} \frac{h_c}{2} \Rightarrow$$ **Neutral axis in the steel profile**

$$A_a(h - z_a - h_c) = 9730\,mm^2(370\,mm - 135\,mm - 86.4\,mm) = 1.45 \cdot 10^6 mm^3$$

$$\frac{A_c}{n_{el,court}} \frac{h_c}{2} = \frac{86.4\,mm \cdot 1275\,mm}{6.1} \frac{86.4\,mm}{2} = 0.78 \cdot 10^6\,mm^3$$

$$A_b = A_a + \frac{A_c}{n_{el,court}} = 9730\,mm^2 + \frac{1275 \cdot 86.4}{6.1} = 27'789 mm^2$$

$$z_b = \frac{1}{A_b} \cdot \left[A_a z_a + \frac{A_c}{n_{el,court}}\left(h - \frac{h_c}{2}\right)\right] = \frac{1}{27789} \cdot \left[9730 \cdot 135 + \frac{1275 \cdot 86.4}{6.1}\left(370 - \frac{86.4}{2}\right)\right] = 259.64\,mm$$

$$I_{b,co}{}^{+} = I_a + A_a(h - z_a)^2 + \frac{1}{3} \frac{A_c}{n_{el,court}} h_c^2 - A_b(h - z_b)^2$$

$$I_{b,co}^{+} = 136.7 \cdot 10^6 + 9730(370 - 135)^2 + \frac{1}{3} \frac{1275 \cdot 86.4}{6.1} 86.4^2 - 27789(370 - 259.64)^2 = 380 \cdot 10^6 mm^4$$

Long load duration: $n_{el,long} = 18.3$

Position of the neutral axis:

$$A_a(h - z_a - h_c) \geq \frac{A_c}{n_{el,court}} \frac{h_c}{2} \Rightarrow$$ **Neutral axis in the steel profile**

$$A_a(h - z_a - h_c) = 9730\,mm^2(370\,mm - 135\,mm - 86.4\,mm) = 1.45 \cdot 10^6 mm^3$$

$$\frac{A_c}{n_{el,long}} \frac{h_c}{2} = \frac{86.4\,mm \cdot 1275\,mm}{18.3} \frac{86.4\,mm}{2} = 0.26 \cdot 10^6\,mm^3$$

$$A_b = A_a + \frac{A_c}{n_{el,long}} = 9730\,mm^2 + \frac{1275 \cdot 86.4}{18.3} = 15'750\,mm^2$$

$$z_b = \frac{1}{A_b} \cdot \left[A_a z_a + \frac{A_c}{n_{el,long}}\left(h - \frac{h_c}{2}\right)\right] = \frac{1}{15750} \cdot \left[9730 \cdot 135 + \frac{1275 \cdot 86.4}{18.3}\left(370 - \frac{86.4}{2}\right)\right] = 208.30\,mm$$

$$I_{b,lo}{}^{+} = I_a + A_a(h - z_a)^2 + \frac{1}{3} \frac{A_c}{n_{el,long}} h_c^2 - A_b(h - z_b)^2$$

$$I_{b,lo}^{+} = 136.7 \cdot 10^6 + 9730(370 - 135)^2 + \frac{1}{3} \frac{1275 \cdot 86.4}{18.3} 86.4^2 - 15750(370 - 208.3)^2 = 277 \cdot 10^6 mm^4$$

Shrinkage: $n_{el,cs} = 12.2$

Position of the neutral axis:

$$A_a(h - z_a - h_c) \geq \frac{A_c}{n_{el,cs}} \frac{h_c}{2} \Rightarrow$$ **Neutral axis in the steel profile**

$$A_a(h - z_a - h_c) = 9730\,mm^2(370\,mm - 135\,mm - 86.4\,mm) = 1.45 \cdot 10^6 mm^3$$

$$\frac{A_c}{n_{el,cs}} \frac{h_c}{2} = \frac{86.4\,mm \cdot 1275\,mm}{12.2} \frac{86.4\,mm}{2} = 0.39 \cdot 10^6\,mm^3$$

$$A_b = A_a + \frac{A_c}{n_{el,cs}} = 9730\,mm^2 + \frac{1275 \cdot 86.4}{12.2} = 18'759\,mm^2$$

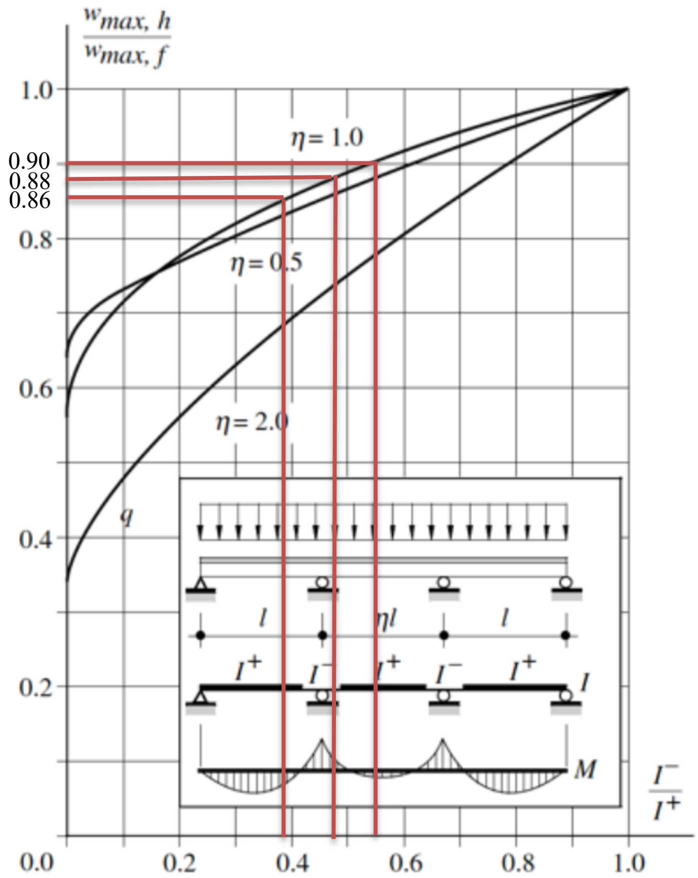
$$z_b = \frac{1}{A_b} \cdot \left[A_a z_a + \frac{A_c}{n_{el,cs}}\left(h - \frac{h_c}{2}\right)\right] = \frac{1}{18759} \cdot \left[9730 \cdot 135 + \frac{1275 \cdot 86.4}{12.2}\left(370 - \frac{86.4}{2}\right)\right] = 227.33\,mm$$

$$I_{b,cs}{}^{+} = I_a + A_a(h - z_a)^2 + \frac{1}{3} \frac{A_c}{n_{el,cs}} h_c^2 - A_b(h - z_b)^2$$

$$I_{b,cs}^{+} = 136.7 \cdot 10^6 + 9730(370 - 135)^2 + \frac{1}{3} \frac{1275 \cdot 86.4}{12.2} 86.4^2 - 18759(370 - 227.33)^2 = 315 \cdot 10^6 mm^4$$

Influence of cracking for computation of deflection

Using the inertia ratios given in Table 2.1, the ratio between the deflection determined for constant inertia can be calculated with the deflection taking into account the influence of cracking on support.



Poutre sur quatre appuis

Checking deflections:
Frequent load case, suitability for operation

$$w = w_2 + w_{31} \le \frac{l}{350}$$
$$w = w(g_{fin}) + w_s + \psi_1 w(q_k)_{court}$$

$w(g_{fin}) = 2.0\,mm$ $\quad \frac{I^-}{I^+} = 0.55 \Longrightarrow \frac{w_h}{w_f} = 0.9$ $\quad \boldsymbol{w(g_{fin}), f} = \frac{2mm}{0.9} = 2.22\,mm$

$\boldsymbol{w_s} = 2.6\,mm$ $\quad \frac{I^-}{I^+} = 0.48 \Longrightarrow \frac{w_h}{w_f} = 0.88$ $\quad \boldsymbol{w_s, f} = \frac{2.6mm}{0.88} = 2.95\,mm$

$\boldsymbol{\psi_1 w(q_k)_{co}} = 2.0\,mm$ $\quad \frac{I^-}{I^+} = 0.39 \Longrightarrow \frac{w_h}{w_f} = 0.39$ $\quad \boldsymbol{\psi_1 w(q_k)_{co}, f} = \frac{2.0}{0.86} = 2.32\,mm$

$\boldsymbol{w, f = 2.22\,mm + 2.95\,mm + 2.32\,mm = 7.5\,mm \le \frac{l}{350} = 17.1\,mm}$ **(+15% deflection)**

Cas de charge fréquent, utilisation et exploitation, confort

$w = \psi_1 w(q_k)_{court} = 0.5 \cdot 3.9\text{ mm} = 2.0\text{ mm} < \frac{l}{350} = 17.1\text{ mm}$ 2.32 mm **(+15% deflection)**

Cas de charge quasi permanent, aspect

$$w = w(g_a + g_b) + w(g_{fin}) + w_s + \psi_2 w(q_k)_{long} \le \frac{l}{300}$$
$$w = 6.4\text{ mm} + 2.0\text{ mm} + 2.6\text{ mm} + 0.3 \cdot 5.4\text{ mm}$$
$$= 12.6\text{ mm} < \frac{6000\text{ mm}}{300} = 20\text{ mm}$$

6.4 + 2.22 + 2.95 + 0.3*5.4/0.9 = 13.4 mm **(+6% deflection)**

Acts on steel cross-section only (constant inertia)